A Literature Review on Extending Defeasible Reasoning Beyond Rational Closure

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ABSTRACT

In artificial intelligence inference is what follows from the knowledge within the system. It is through logical considerations that conclusions or rather inferences made. However, the complexity of this world makes it difficult to infer with certainty since not everything is simply put in black or white. The notion of black and white in logics is found in propositional logics where the truth of one thing strictly implies the truth of the other. This notion fails when the knowledge of the world contradicts itself, and thus giving rise to defeasible reasoning which allows to infer on information that is typical true. It uses the KLM Framework, ranked interpretations and rational closure concept to make a somewhat what accurate inference. The aim of this review is to find points of further optimization beyond rational closure taking into account recent optimization efforts.

CCS CONCEPTS

Theory of computations, Artificial Intelligence, Computing Methodologies

KEYWORDS

Propositional logic, defeasible reasoning, entailment, rational closure, non-monotonicity

Introduction and Motivation

The Reasoning is a very important aspect of artificial intelligence application. It allows machines to make somewhat appropriate decisions or inferences for the context in which they operate. However, due to complexities in the world it often difficult to define logical states or properties of the world or objects within it that do not conflict or that are incomplete or nether true or false [1] . As the number of defined states increases it is more likely the number of conflicts will increase. Now the answer to the question about how to manage this lies in defeasible reasoning. With further investigation in the following topics a broader understanding and basis for extending defeasible reasoning beyond rational closure can be achieved. The topics would be propositional logics (its syntax and semantics) interpretations, satisfiability, Knowledge bases and their states of monotonicity and non-monotonicity, point of failure); Entailment from a classical and defeasible perspective; Defeasible reasoning, The KLM Framework; Ration Closure.

Propositional Logic

Syntax

Propositional logic is a branch of logics that deals with propositions, also known as Atoms. These propositions are words or sentences describing things in the world (e.g., "bird" or "birds fly") and are associated with truth values, that is they can be True or False, in the example of the "bird" this can represent the existence of birds or not and in the case of "birds fly" it can represent the validity of the statement that birds fly [2]. More so propositions can be combined using logical connectives to form more complex propositional statements or to exclude other statements or atoms. The logical connectives being the negation ($\neg$), disjunction ($\vee$), conjunction($\wedge$), conditional($\rightarrow$), and biconditional($\leftrightarrow$). Given that we have atoms α and $\beta\bot$, the negation of α, denoted as $\neg\alpha$ would yield results not in α, i.e. if α is True then $\neg\alpha$ will yield False. The negation is a Unary logical connective since it operates on only one propositional statement or atom at a time. The other logical connectives are binary since they operate on two propositional statements or atoms. The Disjunction of α and β, denoted as $\alpha \vee \beta$, it is read as α or β and evaluates to True if either one of the propositions are True. The Conjunction of α and β denoted as $\alpha \wedge \beta$, it is read as α and β, and evaluates to True if both the propositions are True. The conditional is an implication connective, denoted as $\alpha \rightarrow \beta$, and read as α implies β, that is if α is True then it means β is True. And lastly, the biconditional $\leftrightarrow$) which is the same as the conditional with an added feature of β implying α as well[3], [4].

We can further recursively define a set of propositional statements $\mathcal{L}$; for every p $\in \mathcal{P}$, where $\mathcal{P}$ is set of all atoms; p $\in \mathcal{L}$; and if α, β $\in\mathcal{L}$ then α:{$\neg\alpha$, $\alpha\vee\beta$, $\alpha\wedge\beta$, $\alpha\rightarrow\beta$, $\alpha\leftrightarrow\beta$, $\bot$, $\top$ }, where $\bot$ represents all propositions that are always False, and $\top$

representing all propositions that are always True. The set of all these formulas in $\mathcal{L}$ defines the syntax of propositional logic.

Semantics

Atoms alone do not give much meaning it is their combination using connectives that does. Which brings about the notion of valuations. A valuation u is a function that maps $\mathcal{P}$ to True or False, denoted u:$\mathcal{P}\rightarrow$ {T, F}. It is with the assignment of atoms to Truth values (True or False) that allows us to get meaning of much more complex propositional statements. Suppose we have $\alpha, \beta \in \mathscr{P}$ the assignment of truth values to α and β will yield the results shown in the truth table below (Table 1). Each row in the truth table is a single valuation and collectively can be denoted by $\mathcal{U}$, where $\mathcal{U} = \{\alpha\beta, \alpha\bar{\beta}, \bar{\alpha}\beta, \bar{\alpha}\bar{\beta}\}$, a bar on the alphabet representing its false entry, that is in the second valuation of set $\mathcal{U}$, β is false. With this set or truth table the truth values of more complex propositional statements are found.

Table 1: Valuations for atoms α and β

α	β	¬α	α∨β	α∧β	α→β	α↔β
T	T	F	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	F	T	F	F	T	T

Satisfiability:

An atom satisfies a valuation if the atom is true for that valuation, denoted $u \Vdash \alpha$. where $u \in \mathcal{U}$ such that $u(\alpha) = T$ for the atom alpha. Should u not satisfy the valuation, that is the atom is false for that valuation it is then denoted as follows, $u \nVdash \alpha$. Note that if there are no valuations of α then α is unsatisfiable. This can be extended to compound or complex propositional statements as in table, i.e., in order for $u \vDash \alpha \rightarrow \beta$, α must be false ($u \nVdash \alpha$) or β must be true ($u \Vdash \beta$). Furthermore, all valuations where α is True are regarded to as the models of α.

Entailment:

Entailment is the notion that using the relationship between two or more statements one can deduce more information. Given that we have propositional statements or formulas $\alpha, \beta \in \mathcal{L}$, α is said to entail β, denoted $\alpha \vDash \beta$ if for every valuation $u \in \mathcal{U}$, $u \Vdash \alpha$ and $u \Vdash \beta$. Then we can say that the models of α are a subset of the models of β ($\alpha \subseteq \beta$) if $\alpha \vDash \beta$ and furthermore that β is a logical consequence of α.

Knowledge base:

A knowledge base (denoted by **K**) is a set of propositional statements or formulas that can be used to draw a conclusion, for instance, the set {"birds can fly", "Robins are birds"} can be used to draw a conclusion that "Robins can fly". A knowledge base only holds a subset of a language and not the entire language. Hence, proper generation of the knowledge base whether context based or general is of key interest. If we have a knowledge base **K** and a formula α we can deduce that $K \vDash \alpha$ (**K** entails α) for every $u \in \mathcal{U}$ such that $u \Vdash \mathbf{K}$ and $u \Vdash \alpha$, that is models of **K** are a subset of the models of α.

Monotonicity and Non-monotonicity:

Knowledge bases can be Monotonic or Non-monotonic. Monotonic Knowledge bases are those in which the addition of new formulas or propositional statements does not change the conclusions that were made using the previous knowledge base and but rather contributes by adding more conclusions to possible queries. On the other hand, if an addition of a formula changes conclusions made from previous knowledge base or contradicts with some other formulas or statements that was originally in the knowledge base then the Knowledge base is non-monotonic.

Non-monotonic Knowledge bases consist of statements or formulas that are not always True for some instances and cannot be used in classical reasoning to deduce given a query and as such defeasible reasoning was introduced[5].

DEFEASIBLE REASONING

The issue with propositional logic is that it does not have any mechanisms to handle inconsistencies in knowledge bases. For instance, In the case where new statements are added to the set previously defined above {"birds can fly", "Robins are birds", "penguins are birds", "penguins cannot"} a conflict between statements arises since penguins are birds but do not fly. This results with the penguin considered as not existent and the result is false.

THE KLM FRAMEWORK

The KLM Framework named after Kraus, Lehmann and Magidor introduces this concept of typicality to accommodate formulas or propositional statements that are not always trues [3],[6]. This eliminates the contradiction that would have otherwise been detected under classical reasoning. Typicality is denoted as $|\!\sim$, and replaces the implies or conditional operator (→) in cases there the statements does not always hold like "birds fly" to "birds typically fly". If b and f were to represent bird and fly, respectively. The formula would be translated from a classical reasoning point of view $b \rightarrow f$ to a defeasible one, $b \mid\!\sim f$.

Entailment in defeasible reasoning is denoted by $|\!\approx$, and there no specific methods to determine if defeasible entailment is satisfied. In response to that the KLM proposed properties that must be satisfied in order for the defeasible entailment to hold. These are referred to as the LM – Rational, and are as follows:

Reflexivity:

$$\mathbf{K} \mid\!\approx \alpha \mid\!\sim \alpha$$

Left logical equivalence:

$$\frac{\alpha \equiv \beta, \mathbf{K} \mid\approx \alpha \mid\sim \gamma}{\mathbf{K} \mid\approx \beta \mid\sim \gamma}$$

Right weaking:

$$\frac{\mathbf{K} \mid\approx \alpha \mid\sim \beta, \beta \vDash \gamma}{\mathbf{K} \mid\approx \alpha \mid\sim \gamma}$$

And:

$$\frac{\mathbf{K} \mid\approx \alpha \mid\sim \beta, \mathbf{K} \mid\approx \alpha \mid\sim \gamma}{\mathbf{K} \mid\approx \alpha \mid\sim \beta \wedge \gamma}$$

Or:

$$\frac{\mathbf{K} \mid \approx \alpha \mid\sim \gamma, \mathbf{K} \mid \approx \beta \mid\sim \gamma}{\mathbf{K} \mid \approx \alpha \vee \beta \mid\sim \gamma}$$

Cautious Monotonicity:

$$\frac{\mathbf{K} \mid \approx \alpha \mid\sim \beta, \mathbf{K} \mid \approx \alpha \mid\sim \gamma}{\mathbf{K} \mid \approx \alpha \wedge \beta \mid\sim \gamma}$$

Rational monotonicity:

$$\frac{\mathbf{K} \mid \approx \alpha \mid\sim \gamma, \mathbf{K} \mid \approx \alpha \mid\sim \neg\beta}{\mathbf{K} \mid \approx \alpha \wedge \beta \mid\sim \gamma}$$

Ranked interpretations:

Ranked interpretations are an example of preferential interpretations. Preferential interpretation enforces ordering for valuations, such that the preferred valuations are seen as more typical[7]. In a ranked interpretation a rank is assigned to every possible valuation of a set of atoms $\mathscr{P}$in order of typicality, where the most typical formulas or worlds are assigned a rank of 0. The rank number ranges from 0 to *n*, where *n* is some finite value where all ranks are not empty. There also exist an addition rank called the infinity rank in which all impossible worlds are stored.

Formerly this can be written as follows: A ranked interpretation R is a function that maps $\mathscr{U}$ to $\mathbf{N} \cup (\infty)$, where **N** represents all ranks from 0 to *n*, and satisfies a convexity property for every *i* $\in\mathbf{N}$, if $u \in \mathscr{U}$ such that R(u) = *i*, then there must be a $v \in \mathscr{U}$ such that R(v) = *j* with $0 \leq j < i$. Given R as ranked interpretation and $\alpha,\beta \in \mathcal{L}$, R satisfies $\alpha \mid\sim \beta$ in the most typical rank where both α and β holds, then R is said to be a ranked model of $\alpha \mid\sim \beta$.

RATIONAL CLOSURE

Rational closure in defeasible reasoning is the process of drawing up a conclusion using explicit and implicit statements in the knowledge base. The statements can be ranked and accessed in order of importance when drawing the conclusion. Furthermore, it is a non-monotonic entailment defined by LM [8] and a defeasible entailment that satisfies the LM- Rational described above. In rational closure members of the world only inherit the properties that are most typical unlike the lexicographic closure which selects the most preferred possible worlds, based on their degree of normality. What is meant by most typicality is that given we know "birds fly" and "penguins are bids", penguins do not get to inherit the properties of the world because they are not like typical birds.

There are two types of rational closure, the minimal ranked entailment, and the algorithm. The minimal ranked interpretation defines a partial ordering of all ranked interpretations of **K**, denoted by $\leq$**K**. $R_1 \leq R_2$ if for every $u \in \mathscr{U}$, $R_1(u) \leq R_2(u)$. The partial ordered set has a minimal interpretation for a knowledge base K, denoted $\mathbf{R}^{\mathbf{K}}_{\mathbf{RC}}$. The minimal ranked entailment ($\mid\approx$) can be defined as follows: given a defeasible knowledge base K and a minimal ranked interpretation $\mathbf{R}^{\mathbf{K}}_{\mathbf{RC}}$, for any defeasible implication $\alpha \mid\sim \beta$ if and only if $\mathbf{R}^{\mathbf{K}}_{\mathbf{RC}}$ satisfies $\alpha \mid\sim \beta$.

The algorithm on focuses on the concept of materialization, which is the conversion of defeasible statements to their classical counterparts (e.g., $\alpha\mid\sim\beta$ to $\alpha\rightarrow\beta$). Given a knowledge base defeasible knowledge base **K**, its materialized counterpart is denoted as $\vec{\mathbf{K}}$. The knowledge base $\vec{\mathbf{K}}$ results with formulas as $\alpha\rightarrow\beta$ ($\alpha\rightarrow\beta \in \vec{\mathbf{K}}$) for every $\alpha \mid\sim \beta$ that is in **K** ($\alpha \mid\sim \beta \in \mathbf{K}$). A propositional statement is said to be an exception in K if $\mathbf{K} \vDash \neg\alpha$. This implying that α is typically false in every ranked model.

Rank calculations:

There is an algorithm called the base bank that separates the formulars in the knowledge base into ranks, let us assume that we have defeasible knowledge K with the set of formulas {p→b, b |~ f, p→¬f} to help us illustrate the process, where this knowledge base follows from the contradiction of the penguin not being able to fly even though it is a bird, and b, p and f stands for bird, penguin, and the flying capability.

The base Bank outputs a set of classical implications which are counterparts of the defeasible statements in K. How it does this is by separating the formulas from K into defeasible statements or formulas (K_D) from classic propositional logic statements or formulas (K_C). The results of KD and KC in the context of our example would be K_D = {b |~ f} and K_C = {p→b, p→ ¬f}. The counterparts in K_D correspond to a sequence of exceptional subsets, denoted $\varepsilon_n{}^K$. The material counterpart of the defeasible formulas is denoted as $\overrightarrow{K_D}$ is placed in $\varepsilon_0{}^K$, then for each propositional formula in $\varepsilon_0{}^K$ we check if the union of K_C and $\varepsilon_0{}^K$ satisfies ¬α ($K_C \cup \varepsilon_0{}^K \Vdash \neg\alpha$), that is α is false, where α is the antecedent of the positional statement. If so then the antecedent is α is exceptional with regards to the knowledge base and all information (formulas or propositional statements) in $\varepsilon_0{}^K$ should be moved to the next sequence of exceptional subset, $\varepsilon_1{}^K$. Note that this process is continuous until we reach such a point *m* where the subset $\varepsilon_m{}^K$ is an empty set.

Rank 0 corresponds to $\varepsilon_0{}^K$ and all propositional statements in $\varepsilon_0{}^K$ are stored in Rank 0. The propositional statements that are in K_C are assigned a Rank ∞. Now looking at our example $\overrightarrow{K_D}$ becomes {b→f}, and $\varepsilon_0{}^K$ = {b→f}, then $K_C \cup \varepsilon_0{}^K$ satisfies ¬b, and as such b is an exceptional antecedent belong to Rank 0 (see table below for resulting rank).

Table 2: Ranking of the provided example.

0	b→ f
∞	p→b, p → ¬ f

The Rational closure Determines if a statement or formula is defensibly entailed by the knowledge base. What it does it take in a defeasible implication along with acknowledge base and outputs True if and only if $\alpha \mid\sim \beta$ is in the rational closure of K, that is $K \vDash \alpha \mid\sim \beta$. For $\alpha\mid\sim\beta$ the rational closure checks if the negation of the antecedent is in the counter part for K, denoted $\vec{K} \vDash \neg\alpha$. If this is the case, then α is exceptional and ranks of the exceptional statements are continually removed from the knowledge base.

Otherwise, if that is not the case α is not exceptional and then check if the counter part of α|~β (which is α→β) is entailed by the statements in the current rank. Note that an empty K does not entail any propositional statements.

CURRENT OPTIMIZATION TECHNIQUES

Bonatti, Petrova, and Sauro introduced optimization to the Rational closure by using an optimistic evaluation which exploits incremental reasoning and by using a module extractor for defeasible implications, which its purpose is focusing reasoning on a relevant subset of the knowledge base. The proposed optimization techniques iterate the module extractor. The module extractor is utilized to identify and extract relevant modules or components from a larger knowledge base, it continuously does so by iteration. This reduces computational overhead and enhances performance in handling defeasible implications.t [9].

Contextualized Knowledge Repository (CKR). The CRK framework is introduced to model knowledge bases with two layers a global and local context. The global contains meta-knowledge, defines properties of local contexts, and holds context-independent object knowledge which is shared across all local contexts. Bozzato, Eiter, and Serafini have improved the CKR management of context-dependent knowledge by allowing for the inclusion of defeasible statements or formulas in the global context. The framework now accommodates exceptions to the general object knowledge at the local level.[10]

Bonatti introduced the concept of stable rational closure. The stable rational closure approach to optimizing rational closure in description logics is flexible and optimizes by refining the exceptionality criterion aligned with KLM Framework. Hence, the approach accommodates description logics without strict adherence to the disjoint model union property. The algorithms suggested for reasoning with ranked models can effectively enumerate stable rankings for all defeasible knowledge bases, and thus optimizing the reasoning processes [11].

DISCUSSIONS

Rational closure is not the perfect solutions for solving the contradictions in the knowledge base because the larger the knowledge base the more cases of typicality arise and the supplication of rational closure becomes complex. Another issue is that rational closure does not logical systems. Hence improvement in this regard is require. The improvements can be through refining the exceptionality criteria as Bonatti suggests, making the application of rational closure broader since it is typically applied to description logics that satisfy certain properties, such as the disjoint model union property. This can be achieved by relaxing the constraints, another could be exploring alternative semantics or interpretations of rational closure to better handle complex or uncertain knowledge. Ensuring that the rational closure algorithms are more efficient and possibly integrating rational closure with other reasoning techniques such as probabilistic reasoning or fuzzy logic could improve the framework. Since optimization is the main aim of this particular project further understanding of the optimization techniques mentioned above would be the way forward.

CONCLUSIONS

As explained before reasoning is a very important aspect of as it allows machines to make somewhat appropriate decisions. However, in complex worlds deductions may be wrong due to conflicting information. In such cases classical or propositional reasoning produce incorrect results due to the non-monotonic state of the information or knowledge base. A non-Monotonic knowledge base is that in which additional of new propositional statements or formulas (these are combinations of atoms using connectives) invalidates conclusions or inferences made before. Defeasible reasoning is another concept that comes into play to solve the inference issues in non-monotonic knowledge base using the KLM Framework which introduces this concept of "typicality" to accommodate formulas or propositional statements that are not always trues. The use of Ranked interpretations is an were explored in the context of reason reasoning to enforce ordering for valuations, such that the preferred valuations are seen as more typical. Furthermore Rational Closure which is the process of drawing up a conclusion using explicit and implicit statements in the knowledge base was explored to help give an understanding of how it works so points of optimization can be realized.

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